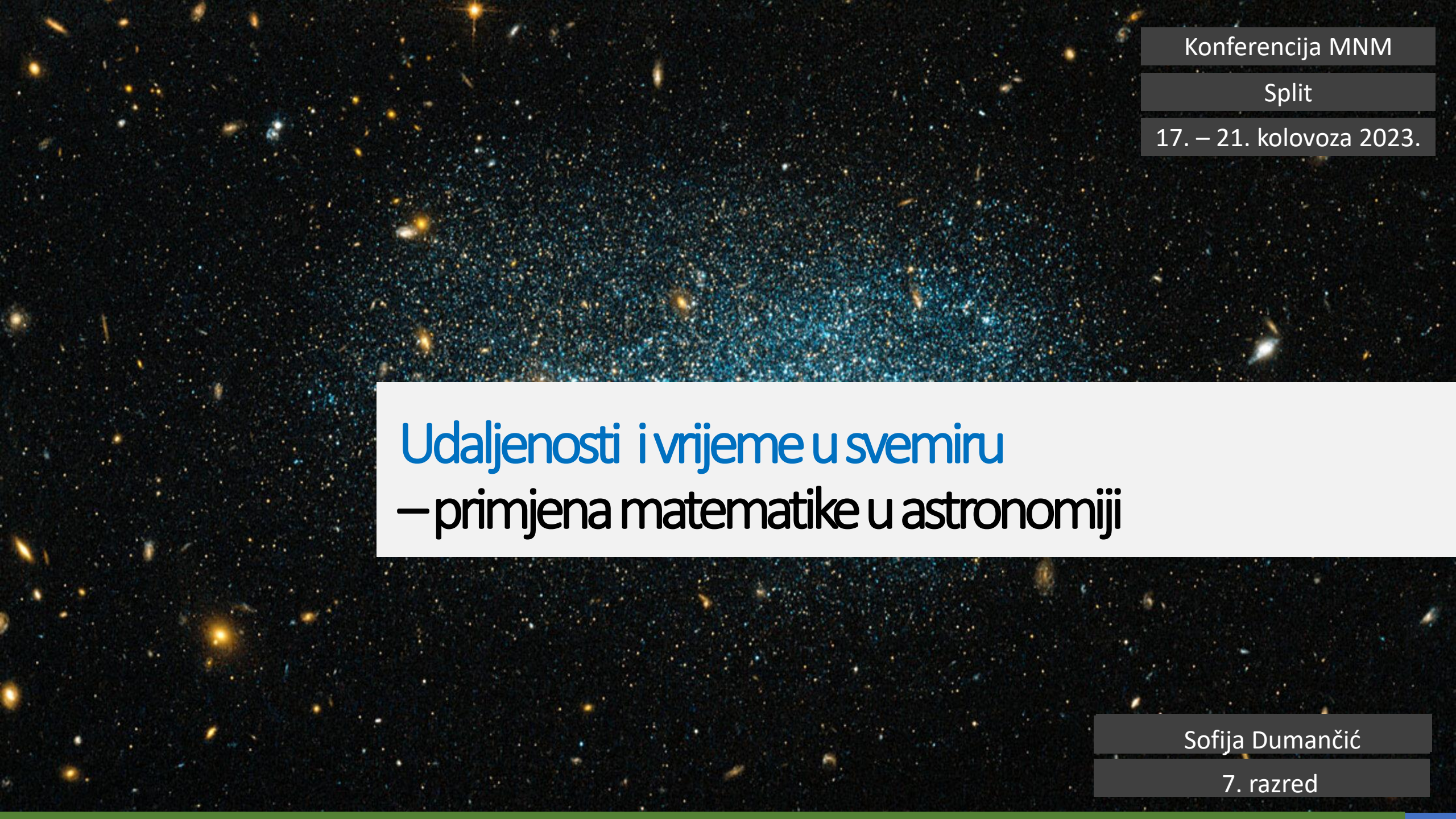


Konferencija MNM

Split

17. – 21. kolovoza 2023.

Udaljenosti i vrijeme u svemiru
– primjena matematike u astronomiji

Sofija Dumančić

7. razred

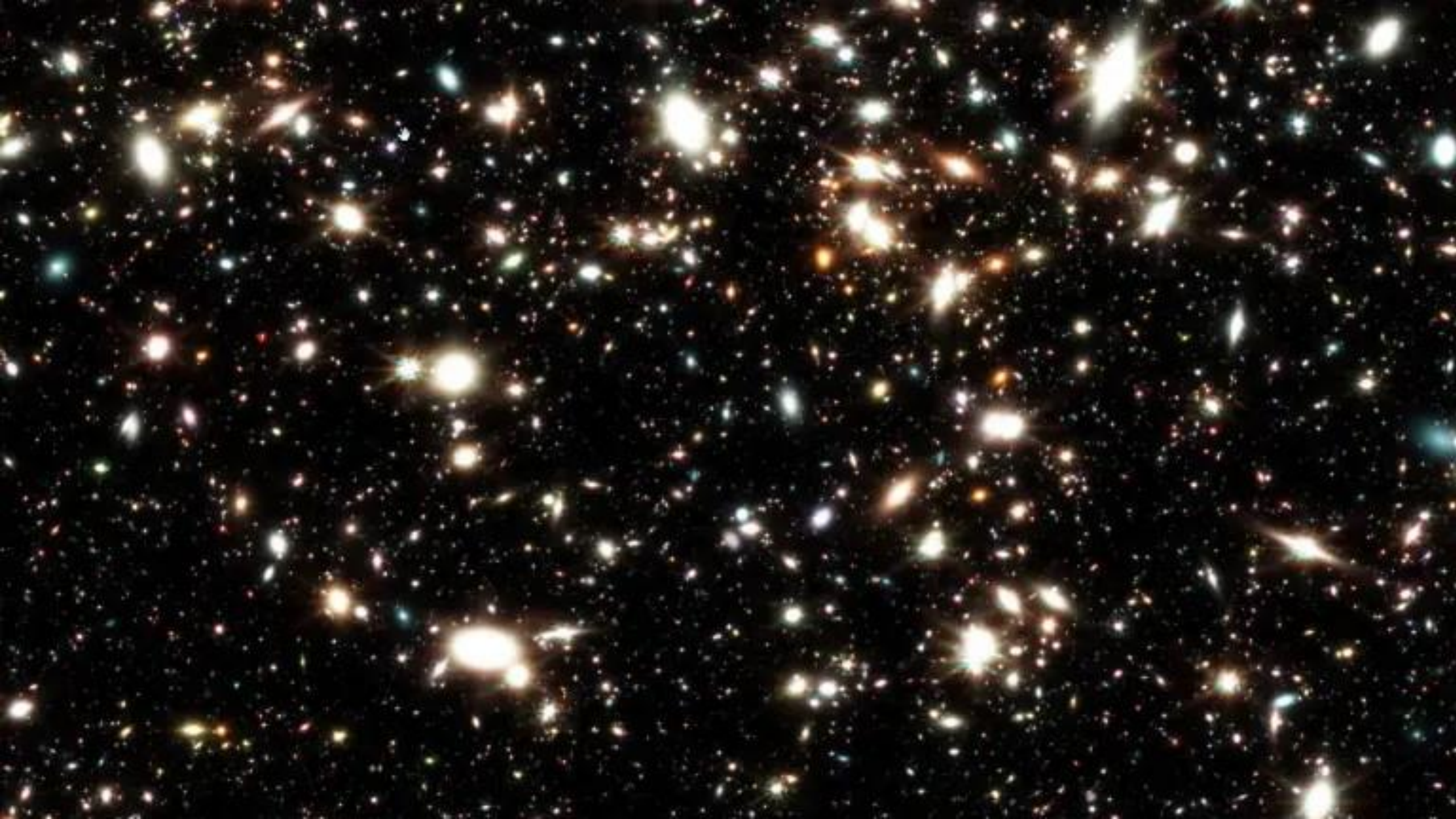
Motivacija

Što je astronomija

- grč. ἀστρονομία: zvjezdoznanstvo
- znanost o svemirskim tijelima i pojavama u svemiru te o njegovu ustroju
- jedna od najstarijih ljudskih djelatnosti
- razvila se iz praktičnih potreba


M 42, M 43, NGC 1977 (Orion)

Maglica Konjska glava



Prikaz Mliječne staze, Kumova slama

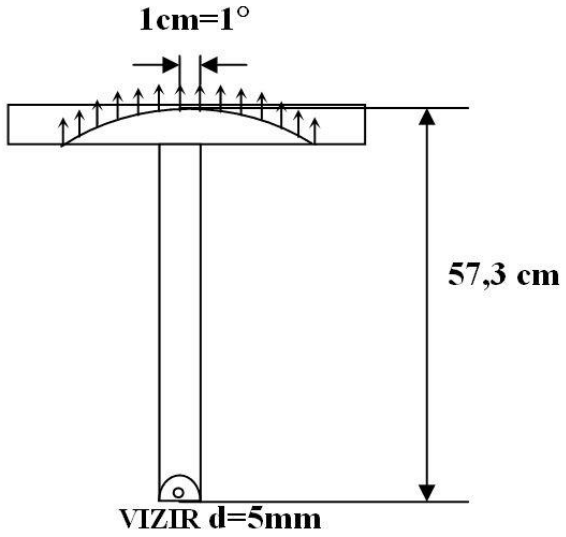

Mliječna staza, Mliječni put, Kumova slama

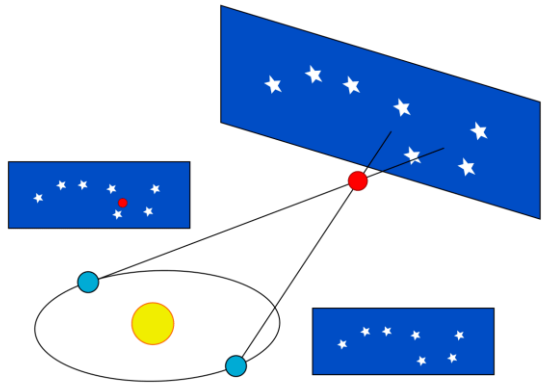
Andromedina galaksija, M 31 (Andromeda)

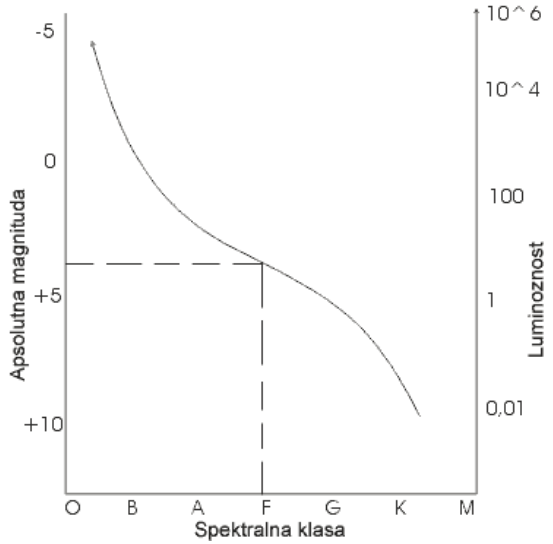
Prikaz sudara Mliječne staze i Andromedine galaksije

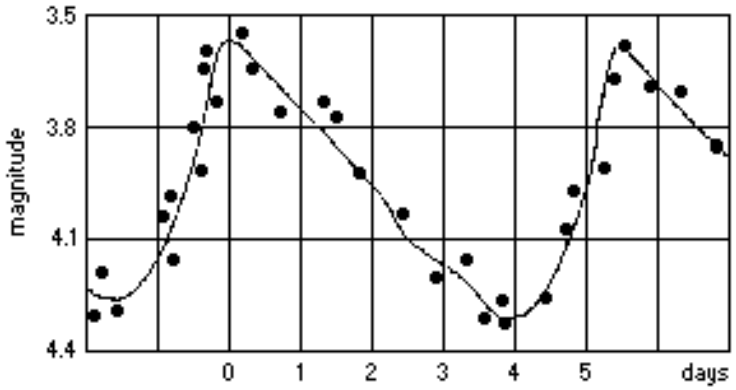
Cilj rada → Odrediti udaljenost galaksije M100

Metode određivanja udaljenosti u astronomiji







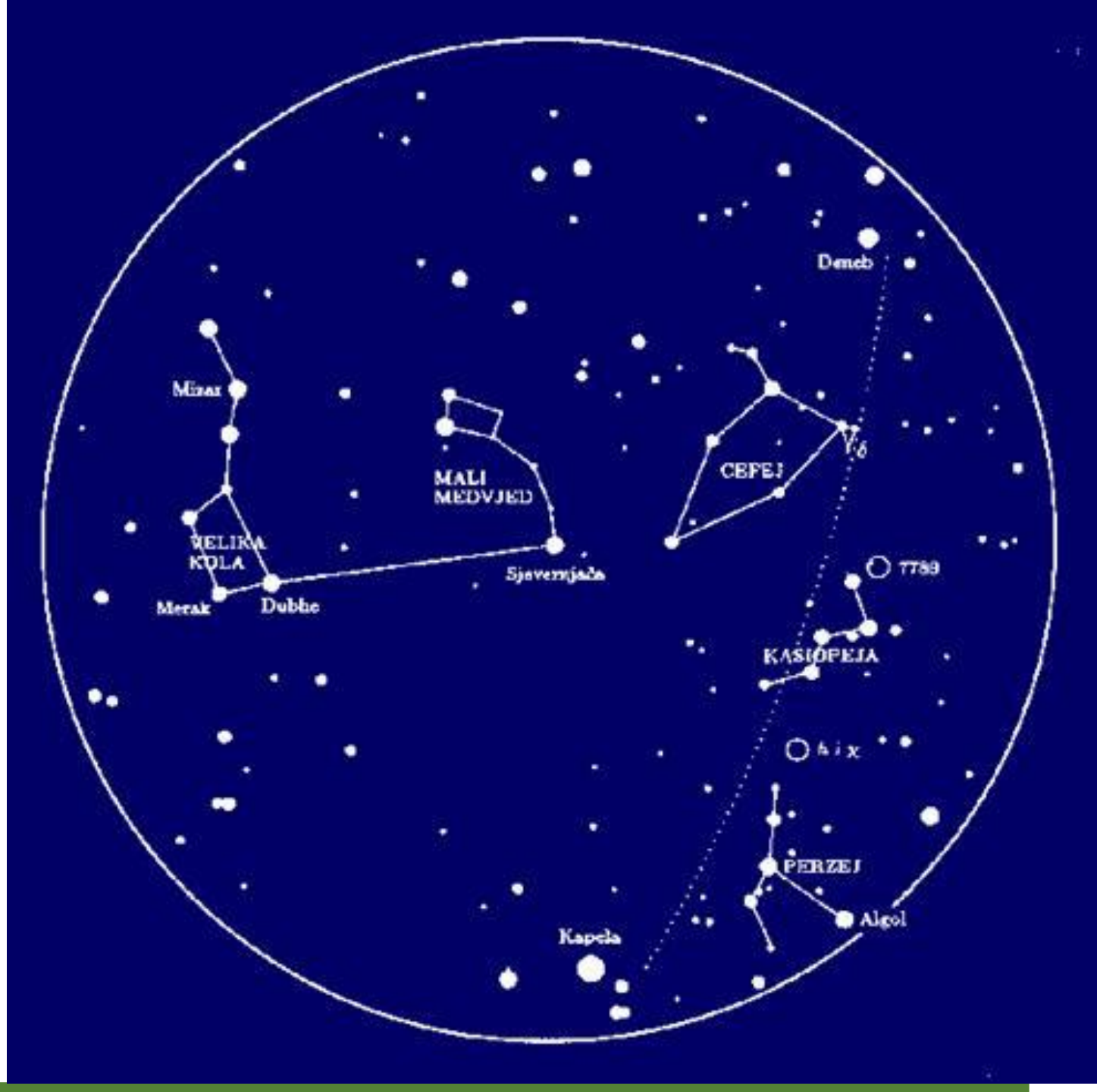


- Astronomske grablje (kutne udaljenosti)
- Metoda zvjezdane paralakse

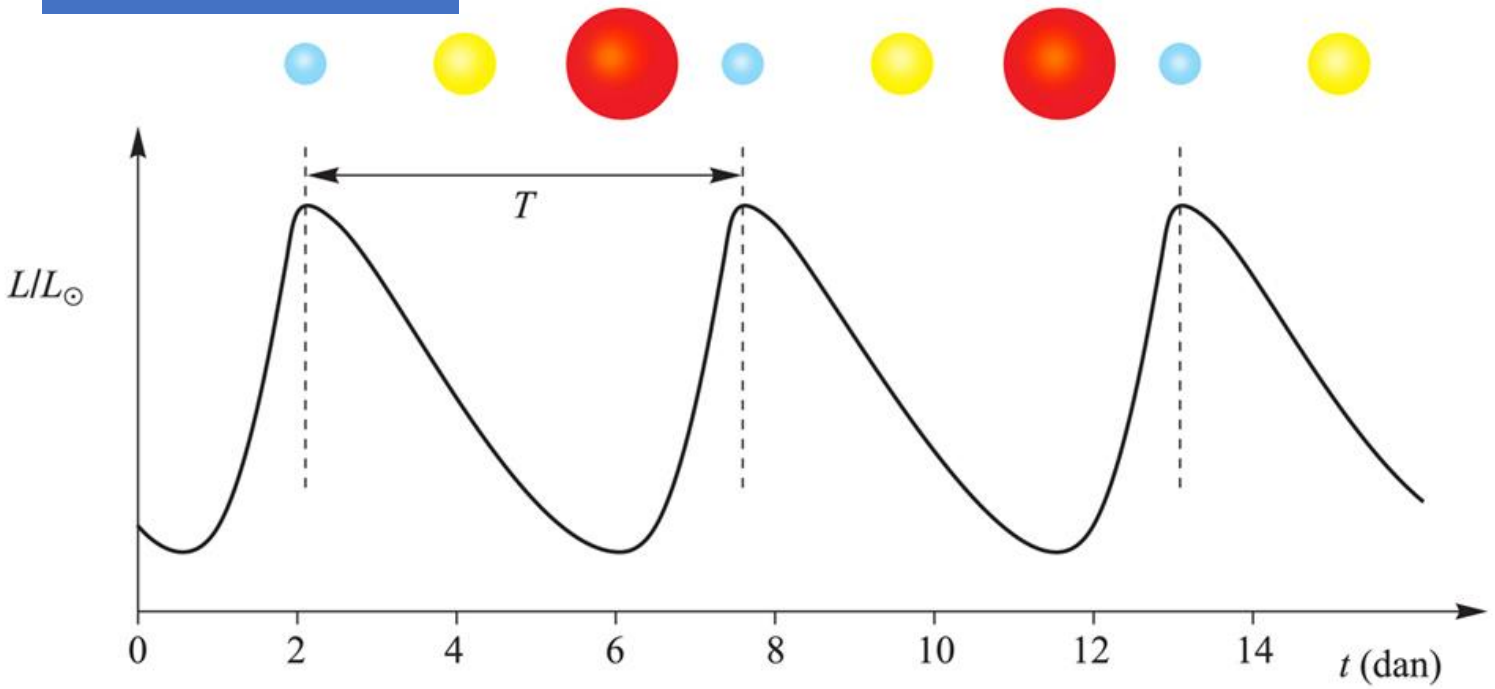
- Metoda spektroskopske paralakse
- Korištenjem promjenjivih zvijezda cefeida
- ...

Promjenjive zvijezde cefeide

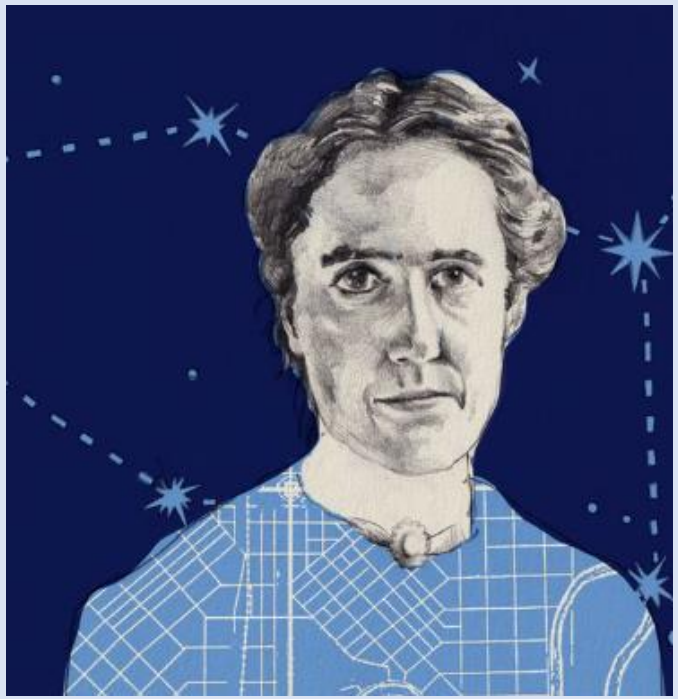
- mlade, sjajne, rijetke,
- karakteristična krivulja sjaja
- **Delta Cefeja**
  - prva zvijezda te vrste
  - otkrivena 1784.
  - 1000 do 100 000 puta sjajnija od Sunca



Promjenjive zvijezde cefeide



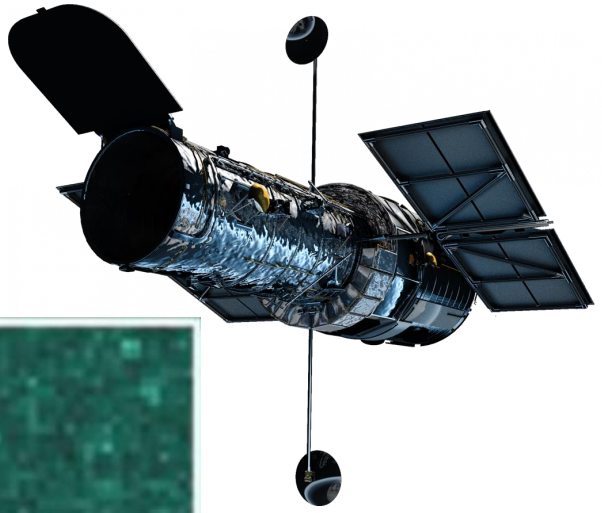
- sjaj se pravilno i periodično mijenja u skladu s promjenom volumena i površinske temperature
- pravilo po kojem se iz perioda sjaja može izračunati apsolutna magnituda promjenjive zvijezde cefeide

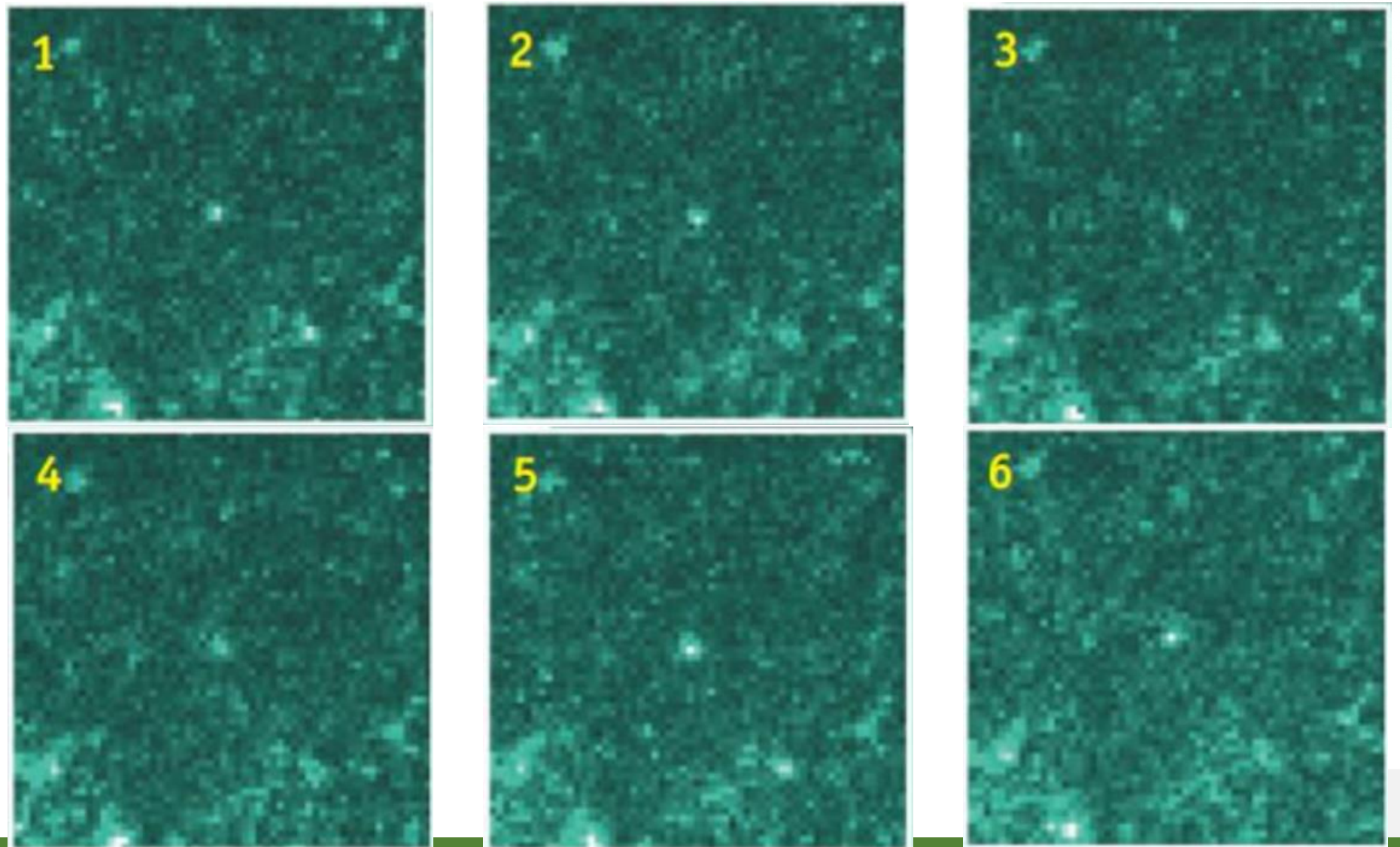


Henrietta Leavitt

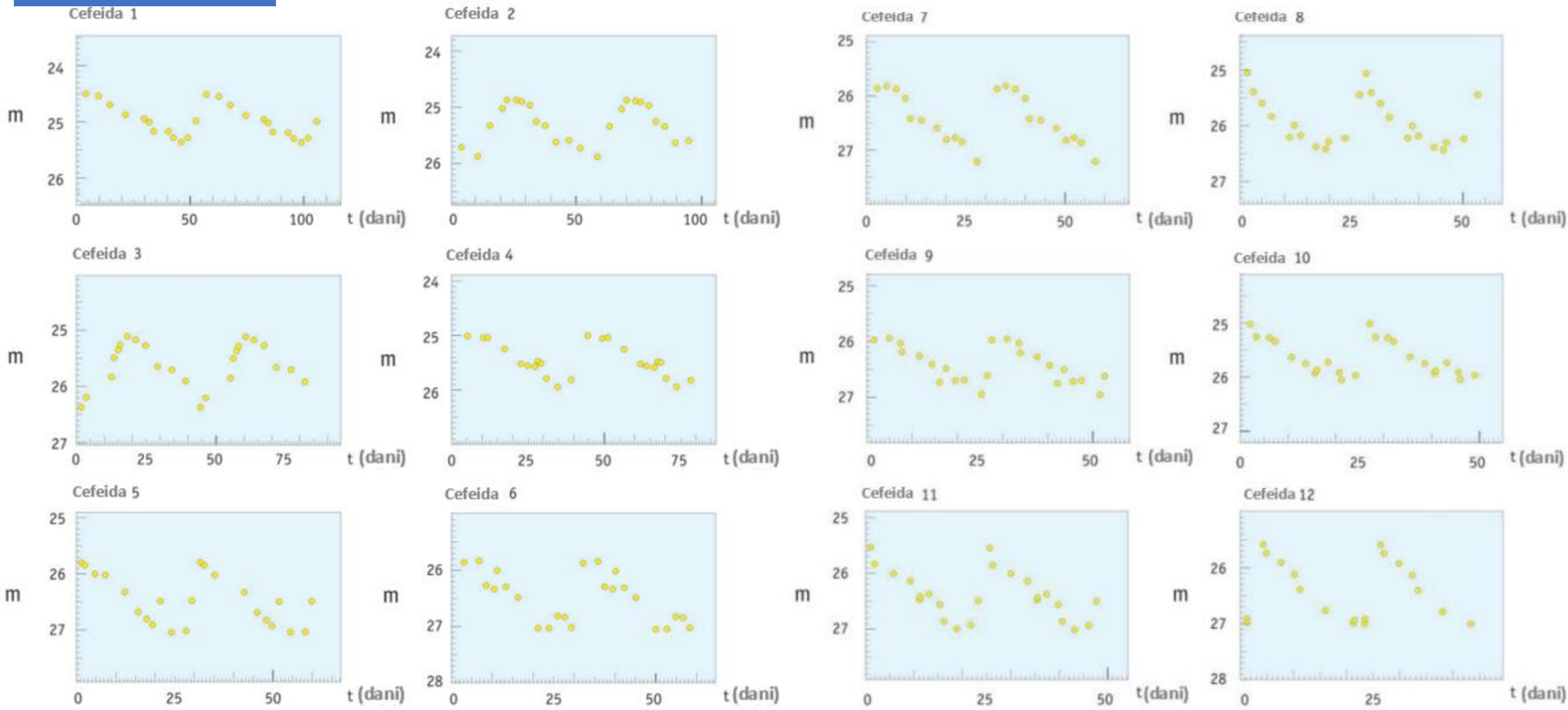
- ✓ sjajnije zvijezde uvijek sporije pulsiraju
- ✓ određivanje udaljenosti zvjezdanih skupova i galaktika s pomoću cefeida

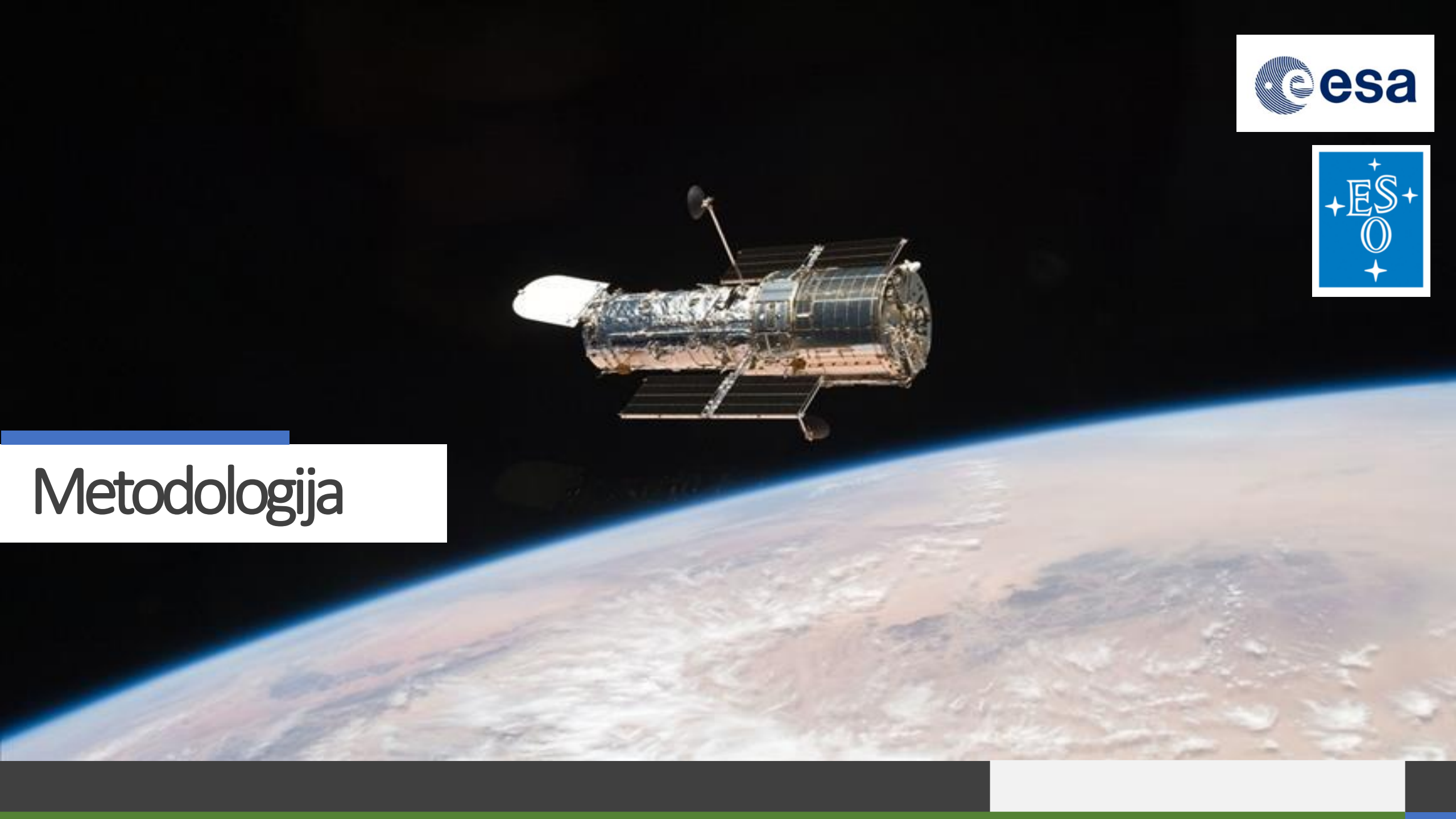
Svemirski teleskop Hubble: niz snimki cefeida iz M 100





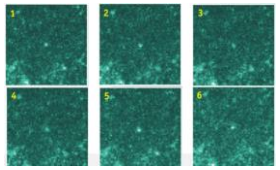
Početak: Krivulje sjaja 12 promjenjivih zvijezda cefeida iz M 100



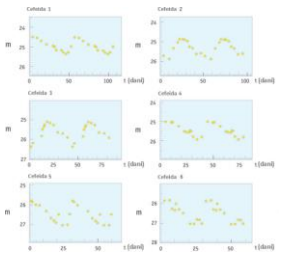


Metodologija

Izračuni udaljenosti za svaku pojedinu cefeidu



Krivulja sjaja

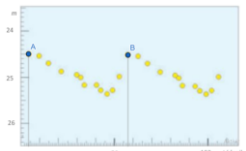


Prividna zvjezdana veličina m

$$m^* = \frac{m_{max} + m_{min}}{2}$$

$$m^{\#} = \frac{zbroj\ svih\ očitanih\ prividnih\ zvjezdanih\ veličina}{broj\ svih\ očitanih\ zvjezdanih\ veličina}$$

Period



Apsolutna zvjezdana veličina M

$M = -2{,}78 \cdot \log P - 1{,}35$

Udaljenost cefeide

$$D = 10^{(m - M + 5):\,5}$$

- 4 vrijednosti za udaljenost svake cefeide

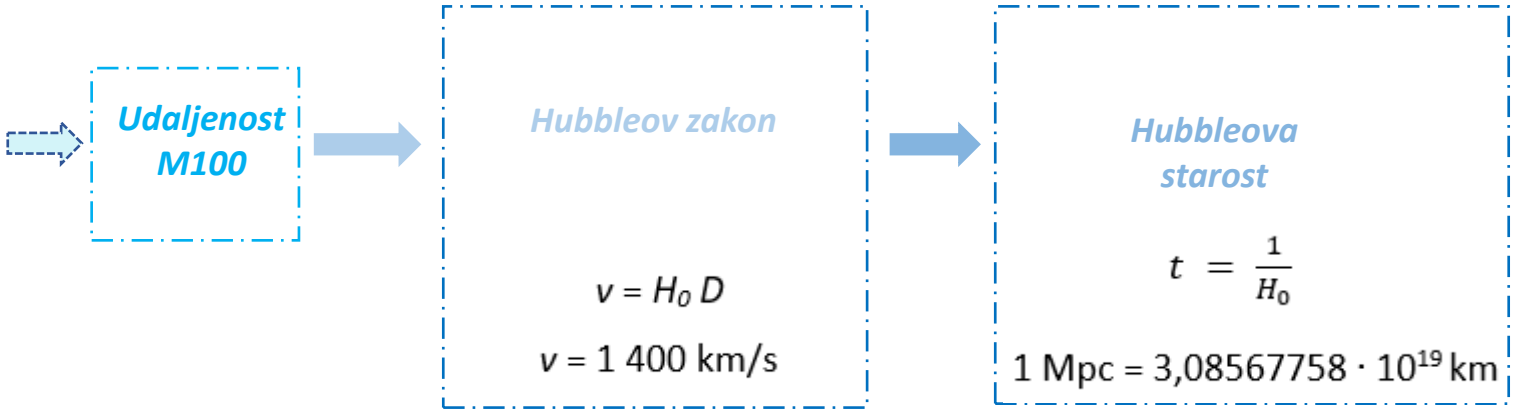
	ručno	digitalno
max i min m	D_r^*	D_d^*
sve vrijednosti m	$D_r^{\#}$	$D_d^{\#}$

Udaljenost M100

Hubbleova konstanta i Hubbleova starost svemira

Edwin Hubble

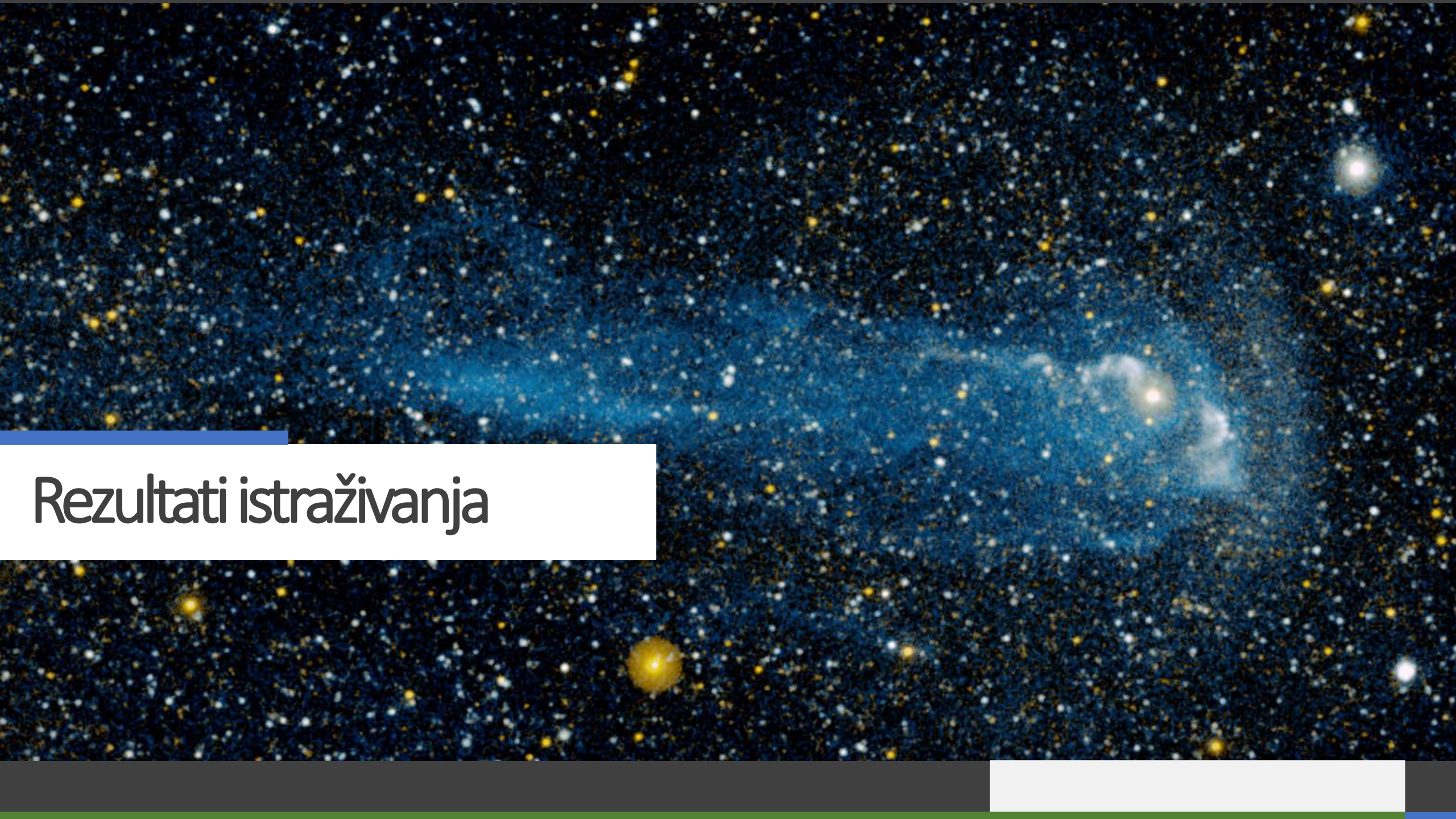
- ✓ veza brzine i udaljenosti galaksija
- ✓ Hubbleova konstanta
- ✓ Hubbleova starost



Dodatni cilj:

Odrediti Hubbleovu konstantu H0 i Hubbleovu starost svemira

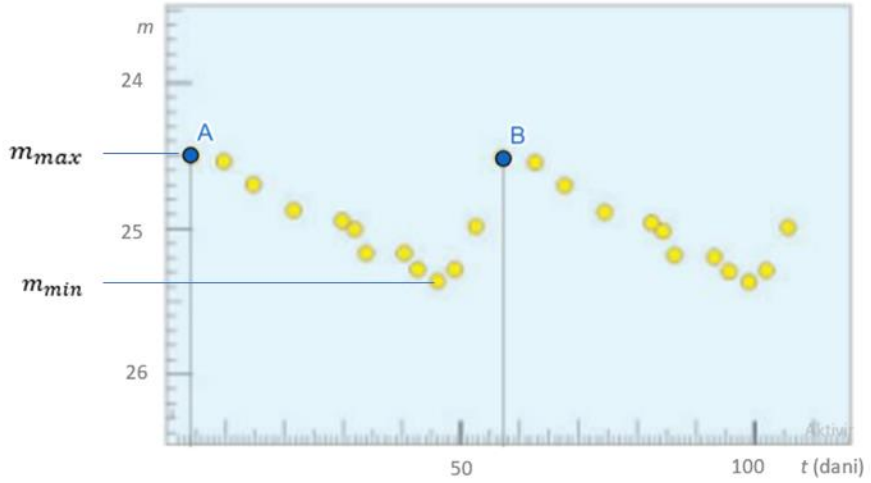
- Svaka od navedenih veličina izračunata je na 4 različita načina



Rezultati istraživanja

Periodi sjaja, apsolutne i prividne zvjezdane veličine 12 cefeida

$$M = -2{,}78 \cdot \log P - 1{,}35$$



$$m^* = \frac{m_{max} + m_{min}}{2}$$

$$m^{\#} = \frac{zbroj\ svih\ očitanih\ prividnih\ zvjezdanih\ veličina}{broj\ svih\ očitanih\ zvjezdanih\ veličina}$$

RUČNO Naziv cefeide	Period *P* (dani)	Apsolutna zvjezdana veličina (*M*)
Cefeida 1	52	–6,1204892952
Cefeida 2	48	–6,0238506400
Cefeida 3	43	–5,8910423065
Cefeida 4	40	–5,8037267759
Cefeida 5	30	–5,4563970881
Cefeida 6	29	–5,4154664342
Cefeida 7	30,5	–5,4763535534
Cefeida 8	26	–5,2836259074
Cefeida 9	25,5	–5,2601817016
Cefeida 10	25	–5.2362732241
Cefeida 11	25	–5.2362732241
Cefeida 12	22	–5,0819350527

DIGITALNI ALATI Naziv cefeide	Period *P* (dani)	Apsolutna zvjezdana veličina (*M*)
Cefeida 1	52,820246160589000	–6,139385169929350
Cefeida 2	47,412725424711000	–6,008987879204580
Cefeida 3	42,127787814146000	–5,866300859087380
Cefeida 4	39,162187924958000	–5,778170112950090
Cefeida 5	30,051641006122000	–5,458473574206330
Cefeida 6	29,201478330491000	–5,423825450327070
Cefeida 7	29,802725837878000	–5,448431645640150
Cefeida 8	26,591621922307000	–5,310790620120370
Cefeida 9	26,099209009297000	–5,288224020050740
Cefeida 10	24,866972707100000	–5,229831711187920
Cefeida 11	24,487450060234000	–5,211263105980840
Cefeida 12	22,006684774745000	–5,082301850915790

RUČNO Naziv cefeide	m_{rmax}	m_{rmin}	m_r^*	$m_r^{\#}$
Cefeida 1	24,5	25,40	24,950	24,96
Cefeida 2	24,9	25,90	25,400	25,59
Cefeida 3	25,1	26,40	25,750	25,29
Cefeida 4	25,0	25,95	25,475	25,43
Cefeida 5	25,8	27,10	26,450	26,45
Cefeida 6	25,2	27,10	26,150	26,42
Cefeida 7	25,8	27,20	26,500	26,38
Cefeida 8	25,1	26,50	25,800	25,87
Cefeida 9	25,9	27,00	26,450	26,35
Cefeida 10	25,0	26,05	25,525	25,61
Cefeida 11	25,5	27,00	26,250	26,35
Cefeida 12	25,6	27,00	26,300	26,35

DIGITALNI ALATI Naziv cefeide	m_{dmax}	m_{dmin}	m_d^*	$m_d^{\#}$
Cefeida 1	24,500000000000000	25,367412691799300	24,93370635	24,952571732266000
Cefeida 2	24,892466362944000	25,890948293930900	25,39170733	25,268322775883600
Cefeida 3	25,097833282622100	26,380634413891400	25,73923385	25,573633154681900
Cefeida 4	24,995195278025000	25,941779826646200	25,46848755	25,414853839261700
Cefeida 5	25,809917633586300	27,033143393573300	26,42153051	26,401445104997900
Cefeida 6	25,847298115343900	27,030478698947700	26,43888841	26,442182705656500
Cefeida 7	25,828249818179400	27,209733960545400	26,51899189	26,384380186555000
Cefeida 8	25,051129434002200	26,400000000000000	25,72556472	25,841425211609600
Cefeida 9	25,949190769784100	26,935410961076800	26,44230087	26,382802608869900
Cefeida 10	24,978458611344400	26,047504728897100	25,51298167	25,575487920127600
Cefeida 11	25,524806145493700	27,000000000000000	26,26240307	26,317752141545000
Cefeida 12	25,578566980062700	27,000000000000000	26,28928349	26,352214877200700

Udaljenosti 12 promjenjivih cefeida

$$D = 10^{(m-M+5):5}$$

$$e = (m - M + 5) : 5 =$$

$$= (24{,}95 - (-6.1204892952) + 5) : 5 =$$

$$= 7{,}2140978591$$

$$D = 10^{e} =$$

$$= 10^{7{,}2140978591} =$$

$$= 16\ 371\ 853{,}850583 \text{ pc}$$

RUČNO Naziv cefeide	e_r*	D_r* (pc)	$e_r^{\#}$	$D_r^{\#}$ (pc)
Cefeida 1	7,2140978591	16 371 853,850583	7,21640555136864	16459079,8190144
Cefeida 2	7,2847701280	19 265 049,444352	7,32323166644237	21049009,6123681
Cefeida 3	7,3282084613	21 291 607,976741	7,23666999976379	17245270,0591637
Cefeida 4	7,2557453552	18 019 608,669312	7,24689920133219	17656279,7526951
Cefeida 5	7,3812794176	24 059 102,234180	7,38051018685490	24016526,0062918
Cefeida 6	7,3130932868	20 563 322,503509	7,36617020990874	23236473,0827617
Cefeida 7	7,3952707107	24 846 814,101411	7,37027071067681	23456905,0922387
Cefeida 8	7,2167251815	16 471 197,758467	7,23057133531793	17004792,4401018
Cefeida 9	7,3420363403	21 980 437,899936	7,32280557109051	21028368,1186601
Cefeida 10	7,1522546447	14 198 898,177311	7,16879310636011	14750036,8855654
Cefeida 11	7,2972546448	19 826 892,176653	7,31648541405242	20724564,5416487
Cefeida 12	7,2763870105	18 896 745,345000	7,28729610144624	19377426,6424644

DIGITALNI ALATI Naziv cefeide	e_d*	D_d*(pc)	$e_d^{\#}$	$D_d^{\#}$ (pc)
Cefeida 1	7,21461830316580	16391485,0962016	7,21839138043907	16534511,9534903
Cefeida 2	7,28013904152840	19060708,5830982	7,25546213101763	18007861,0525243
Cefeida 3	7,32110694146882	20946281,7743180	7,28798680275387	19408268,9927154
Cefeida 4	7,24933153305714	17755443,8420141	7,23860479044237	17322269,4748162
Cefeida 5	7,37600081755722	23768447,6101920	7,37198373584084	23549610,8986920
Cefeida 6	7,37254277149457	23579944,1161948	7,37320163119671	23615743,9390179
Cefeida 7	7,39348470700052	24744843,2471254	7,36656236643903	23257464,4846292
Cefeida 8	7,20727106742429	16116512,4280520	7,23044316634600	16999774,7268166
Cefeida 9	7,34610497709624	22187326,6415584	7,33420532578413	21587647,8893672
Cefeida 10	7,14856267626173	14078703,9393306	7,16106392626311	14489851,2217036
Cefeida 11	7,29473323574554	19712115,5260728	7,30580304950516	20221019,5703615
Cefeida 12	7,27431706818943	18806893,6279261	7,28690334562329	19359910,5165041

Udaljenost M100 i Hubbleova konstanta

$$D = \frac{D_1 + D_2 + \ldots + D_{11} + D_{12}}{12}$$

1 Mpc = 10^6 pc

D = 19 554 048,17805 pc = 19,55404817805 Mpc (dobivena iz e_r*, D_r*),

D = 19 667 061,00441 pc = 19,66706100441 Mpc (dobivena iz $e_r^{\#}$, $D_r^{\#}$),

D = 19 762 392,20267 pc = 19,76239220267 Mpc (dobivena iz e_d*, D_d*),

D = 19 529 494,56005 pc = 19,52949456005 Mpc (dobivena iz $e_d^{\#}$, $D_d^{\#}$).

$$v = H_0 D$$

v = 1 400 km/s

$H_0 = 71{,}5966842753$ km/s/Mpc (dobivena iz e_r*, D_r*),

$H_0 = 71{,}1850133421437$ km/s/Mpc (dobivena iz $e_r^{\#}$, $D_r^{\#}$),

$H_0 = 70{,}8416261372747$ km/s/Mpc (dobivena iz e_d*, D_d*),

$H_0 = 71{,}6864430717856$ km/s/Mpc (dobivena iz $e_d^{\#}$, $D_d^{\#}$).

Hubbleova starost (starost svemira)

$$t = \frac{1}{H_0}$$

1 Mpc = 3,08567758 · 10^{19} km

1 godina = 31 536 000 sekundi

$H_0 = 71{,}5966842753$ km/s/Mpc

$= 2{,}320282207603433114513039929355 8 \cdot 10^{-18}\ s^{-1}$

$t = \frac{1}{H_0} = 430\ 982\ 057\ 580\ 348\ 093{,}70714285714286\ s.$

$t = 13\ 666\ 351\ 394{,}607689425010871928 68$ godina

$t = 13{,}666351394607689425010871928 68 \cdot 10^{9}$ godina,

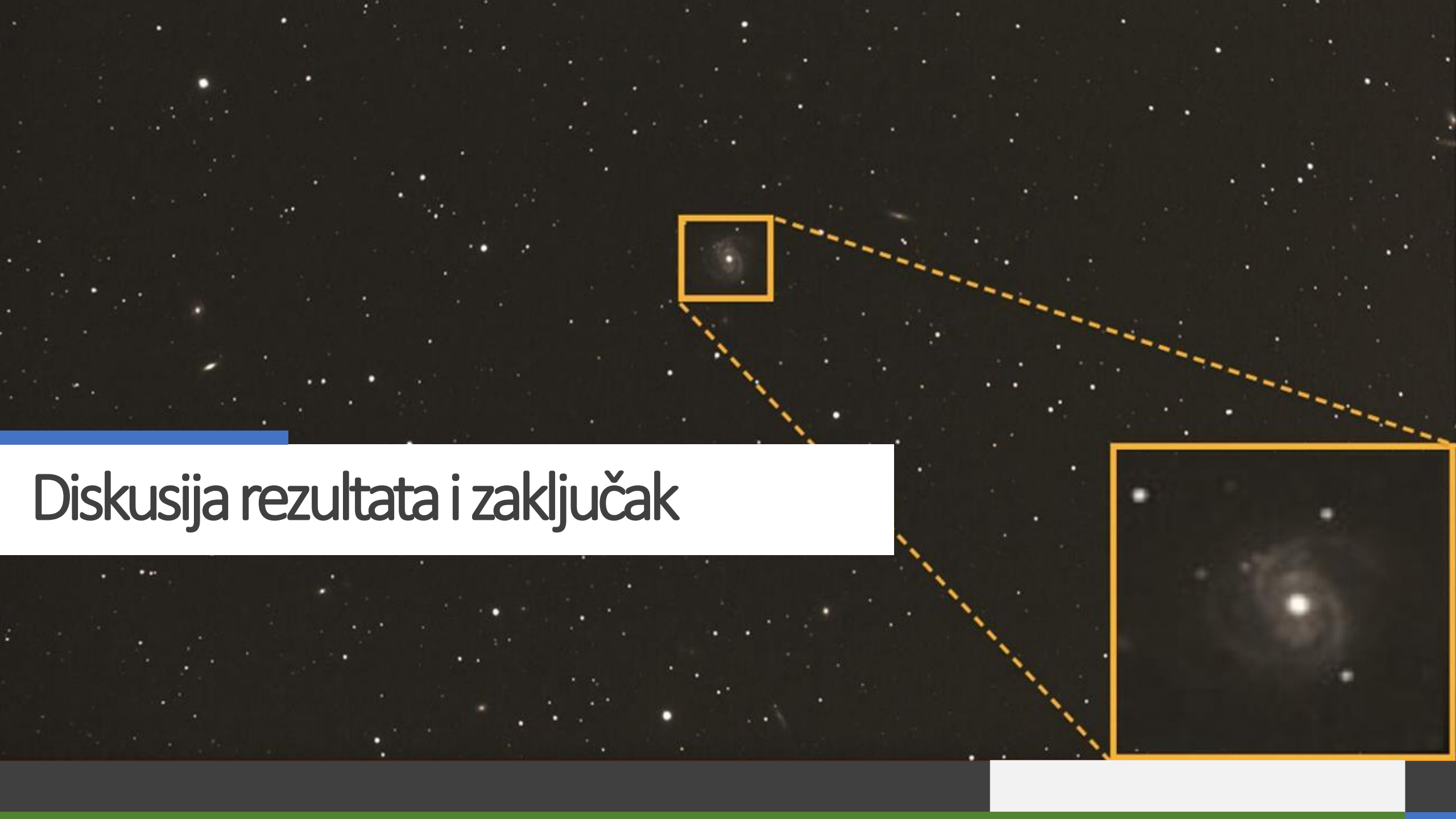
$t \approx$ 13,67 milijardi godina

$t = 13{,}666351595 \cdot 10^{9} \approx 13{,}67$ milijardi godina (dobivena iz e_r^*, D_r^*),

$t = 13{,}745336215 \cdot 10^{9} \approx 13{,}75$ milijardi godina (dobivena iz $e_r^{\#}$, $D_r^{\#}$),

$t = 13{,}811963322 \cdot 10^{9} \approx 13{,}81$ milijardi godina (dobivena iz e_d^*, D_d^*),

$t = 13{,}649190836 \cdot 10^{9} \approx 13{,}65$ milijardi godina (dobivena iz $e_d^{\#}$, $D_d^{\#}$).



Diskusija rezultata i zaključak

Apsolutne i relativne pogreške

Apsolutne pogreške:

$$x = \bar{x} \pm \Delta x_{max}$$

$D \approx 19{,}63 \pm 0{,}1$ Mpc

$H_0 \approx 71{,}33 \pm 0{,}5$ km/s/Mpc

$t \approx 13{,}72 \pm 0{,}1$ milijardi godina

Postupak računanja za D:

D = 19 554 048,17805 pc = 19,55404817805 Mpc (dobivena iz e_r^*, D_r^*),

D = 19 667 061,00441 pc = 19,66706100441 Mpc (dobivena iz $e_r^{\#}$, $D_r^{\#}$),

D = 19 762 392,20267 pc = 19,76239220267 Mpc (dobivena iz e_d^*, D_d^*),

D = 19 529 494,56005 pc = 19,52949456005 Mpc (dobivena iz $e_d^{\#}$, $D_d^{\#}$).

$$\bar{D} = \frac{1}{4}\sum_{i=1}^{4} D_i$$

$$\Delta D_{max} = \max(|D_i - \bar{D}|)\,, \text{za } i = 1, 2, 3, 4$$

Apsolutne i relativne pogreške

Relativne pogreške:

$$r_x = \frac{\Delta x}{\bar{x}} \cdot 100$$

$$r_D \approx 0,5\,\%$$

$$r_{H0} \approx 0,7\,\%$$

$$r_t \approx 0,7\,\%$$

Postupak računanja za D:

D = 19 554 048,17805 pc = 19,55404817805 Mpc (dobivena iz e_r^*, D_r^*),

D = 19 667 061,00441 pc = 19,66706100441 Mpc (dobivena iz $e_r^{\#}$, $D_r^{\#}$),

D = 19 762 392,20267 pc = 19,76239220267 Mpc (dobivena iz e_d^*, D_d^*),

D = 19 529 494,56005 pc = 19,52949456005 Mpc (dobivena iz $e_d^{\#}$, $D_d^{\#}$).

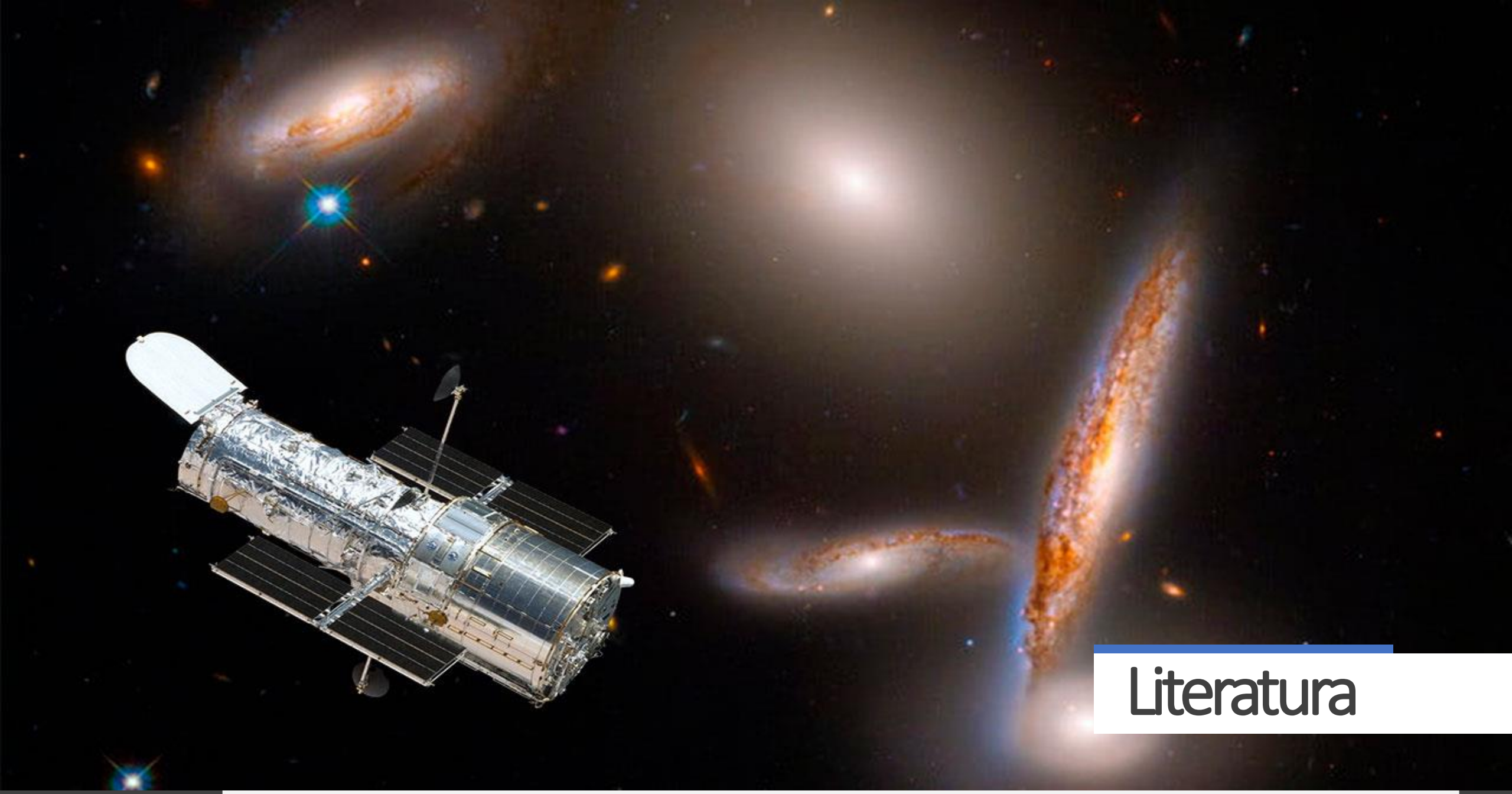
$$\Delta D = \frac{1}{4}\sum_{i=1}^{4}|D_i - \bar{D}|$$

$$r_D = \frac{\Delta D}{\bar{D}} \cdot 100, \text{za } i = 1,2,3,4$$

tj. $$r_D = \frac{\frac{1}{4}\sum_{i=1}^{4}|D_i - \bar{D}|}{\bar{D}} \cdot 100, \text{za } i = 1,2,3,4$$

Usporedbe s literaturom

Veličina	Ovaj rad	Freedman i dr.	ESA i ESO vježba	Drugi rezultati (prema [7])
Period (dani)	22 – 52	20 – 65	22 – 53,5	10 – 60
Prividne zvjezdane veličine	24,5 – 27,20	25,0 – 26,5	25,60 – 27,10	---
Udaljenost M100 (Mpc)	19,63 ± 0,1	17,1 ± 1,8	19,85	27,7 (Sandage) 11,8 (de Vaucouleurs) 15 ± 4 (J.R.M. i dr.) 18,4 ± 2,2 (Pierce, Tully) 14,5 ± 2,7 (Pierce)
Hubbleova konstanta (km/s/Mpc)	71,33 ± 0,5	69 ± 14	70,53	Općenito: 60 – 80
Starost svemira (milijarde godina)	13,72 ± 0,1	---	13,87	Općenito: 12 – 14 (prema [10])



Literatura

Literatura

[1] Vladis Vujnović, Astronomija 1, Osnove astronomije i planetski sustavi, Školska knjiga, Zagreb, 2005.

[2] Heather Couper, Nigel Henbest, Enciklopedija svemira, Znanje, Zagreb, 2004.

[3] Vladis Vujnović, Rječnik astronomije i fizike svemirskog prostora, Školska knjiga, Zagreb, 2004.

[4] Chris Carroll, Oko beskraja, National Geographic, br. 2, 2003.

[5] Margaret Penston, Ian Morison, Astronomija, Leo commerce, Rijeka – Zagreb, 2006.

[6] Marino Fonović, Cefeide, Polaris, http://free-pu.t-com.hr/fonovic/clanci/cefeide_polaris.pdf (Pregledano: 28. 2. 2022.)

[7] Wendy L. Freedman i dr., Distance to the Virgo cluster galaxy M100 from Hubble Space Telescope observations of Cepheids, Nature, Vol. 371, 1994.

[8] Dragan Roša, Metode astronomskih istraživanja, Alfa, Zagreb, 2010.

[9] Dragan Roša, Elementarna astronomija, Zvjezdarnica Zagreb – Zagrebački astronomski savez, Hrvatsko astronomsko društvo, Zagreb, 2020.

[10] The ESA, ESO, The Distance to M100 as Determined by Cepheid Variable Stars, https://sci.esa.int/documents/34439/36575/1567254558082-4-exercise2-high.pdf (Pregledano: 14. 3. 2022.)

[11] eškola, Kozmološko načelo, Hubbleov zakon, Zvjezdarnica Zagreb, https://eskola.zvjezdarnica.hr/osnove-astronomije/kozmologija/kozmolosko-nacelo-hubbleovzakon/ (Pregledano: 14. 3. 2022.)

Hvala na pažnji!